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<p>The purpose of this grant has been to undertake research in the general area of adaptive-grid finite element methods in estimation and control problems. The overall goal has been to investigate adaptive gridding numerical algorithms and to develop a solid theory to handle the mathematical questions of convergence and stability which arise in the use of such algorithms.</p> <p>The P.I. has met this goal through the study of a number of adaptive-grid finite element methods for parameter estimation problems governed by distributed parameter systems. In addition, stability problems encountered in the implementation of such algorithms have led her to make extensive studies of the ill-posed nature of these problems. In particular, theoretical and numerical studies were undertaken concerning regularization schemes for a number of ill-posed estimation problems. Specific applications considered in this latter study include the inverse heat conduction problem and regularizing aspects of descent methods as used to minimize fit-to-data functions associated with linear estimation problems.</p>					
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# FINAL TECHNICAL REPORT

**Grant Number:** AFOSR-89-0419  
**Institution:** Michigan State University  
**Principal Investigator:** Patricia K. Lamm  
**Funding Period:** June 15, 1989 – July 14, 1991

July 15, 1991

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In the area of adaptive finite element methods, the P.I. has made two fundamental contributions. One is the a study of very general conditions required to implement parameter estimation schemes for parabolic and hyperbolic problems when the underlying parameter spaces are parameter-dependent. Such problems arise when the domain itself, or significant features of the domain, are to be estimated; numerical schemes for such problems are typically adaptive in nature because both the domain and domain grid are changing. This project resulted in several papers and talks, detailed below.

A second contribution is a project completed with collaborator **I. G. Rosen** (University of Southern California) concerning the estimation of parameters in degenerate partial differential equations. This work involved differential equations in which the time derivative may degenerate in a certain sense, leading to a set of parabolic equations on one part of the domain and a set of elliptic equations on the remainder of the domain. Such problems arise in many applications, for example, fluids applications; and, because the unknown parameter may actually be the location of the domains for the separate parabolic and elliptic problems, adaptive finite elements are particularly well-suited for use in this problem. This project also resulted in numerous papers and talks, detailed below.

Some preliminary work has been done by the P.I. on adaptive methods for fluids equations, including control and estimation problems associated with an unknown or changing airfoil shape. However, it was quickly ascertained that an understanding of the underlying instability of such ill-posed problems would be required before any reasonable adaptive methods could be implemented. For this reason, the following studies were undertaken concerning the ill-posed nature of parameter estimation problems in general, and for partial differential equation problems in particular.

The P.I. began and continues to work on a project with **J. V. Beck** (Michigan State University) studying stability aspects of the inverse heat conduction problem. In particular, the P.I. has been looking at Beck's "future estimation" method as a type of regularization method. This work, which is both numerical and theoretical in nature, is a collaborative effort with **K. A. Murphy** (University of North Carolina at Chapel Hill). A second stability problem (for parameter estimation in elliptic equations) was begun with collaborator **L. W. White** (University of Oklahoma), although this project is still in its early stages.

Finally, the P.I. has studied the regularization properties of certain descent methods used to minimize fit-to-data functions associated with linear estimation problems. It has been long hypothesized that the stopping criteria in such descent methods could be viewed as a regularization parameter. In work collaborative with **K. A. Murphy**, the P.I. has begun preliminary theoretical and numerical studies on this problem. In particular, we are considering the study of descent methods for applications involving backwards parabolic equations and the inverse heat conduction problem.

#### **Papers Presented and Completed** (not including those works in progress; see above)

- Conference/Workshop Invitations

- "Parameter Estimation for partial differential equations", *Inverse Problems Workshop*, E. Lansing, MI, June 1989.
- "Identification of degenerate distributed parameter systems" (co-authors C. K. Lo, I. G. Rosen), invited minisymposium talk at the *IFAC Symposium on Control of Distributed Parameter Systems*, Perpignan, France, June 1989.
- "Estimation of degeneracies in partial differential equations" (co-author I. G. Rosen), invited minisymposium talk at the *Conference on Decision and Control*, Tampa, FL, December 1989.
- "Regularization and the adjoint method of solving inverse problems, Parts 1,2, and 3", series of invited lectures, at *Inverse Problems in Engineering*, E. Lansing, MI, June 1990.
- "An approximation theory for the estimation of parameters in distributed systems", invited session talk at the *International Conference on Identification in Dynamical Systems and Inverse Problems*, Suzdahl, U.S.S.R., September 1990.

- Colloquia/Seminars on general parameter estimation and ill-posed problems:

- Partial Differential Equations Seminar (2 seminars), Mathematics Department, University of North Carolina at Chapel Hill, April 1990.
- Colloquium, Curriculum in Ecology, University of North Carolina, March 1990.
- Colloquium, Geology Department, Duke University, April 1990.
- "Linear Ill-Posed Problems", semester-long seminar given by the P.I. to faculty and graduate students in the Mathematics Department, University of North Carolina, Spring 1991.

## References

- [1] P. K. Lamm, C. K. Lo, and I. G. Rosen. Identification of degenerate distributed parameter systems. In *IFAC Symposium on Control of Distributed Parameter Systems*, pages 347-352, Perpignan, France, June 1989.
- [2] P. K. Lamm and I. G. Rosen. An approximation theory for the estimation of parameters in degenerate Cauchy problems. *J. Math. Analysis Applic.*, to appear